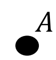
Beginning Constructions – Lines Segments, Circles, and Equilateral Triangles

BEGINNING CONSTRUCTIONS – UNDEFINED TERMS

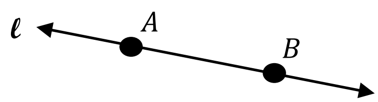
Before we begin creating some basic geometric constructions, we need to get a little bit of vocabulary out of the way. The following concepts are called *undefined terms*, because, honestly, they have no formal definition—they're all an abstract idea that we all agree on.

A **point** is a location that has no dimension. It is represented by a dot.



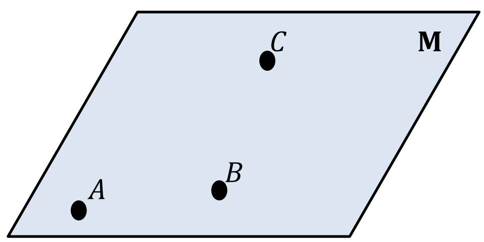
point A

A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end. Through any two points, there is exactly one line. You can use any two points to name the line.



Line ℓ, line AB ($\overleftrightarrow{AB}$), or line BA ($\overleftrightarrow{BA}$).

A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end. Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



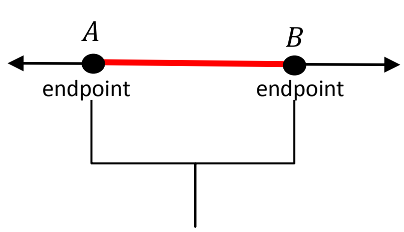
Plane M or plane ABC.

Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

BEGINNING CONSTRUCTIONS – DEFINED TERMS

Now that we have some undefined terms, now we can have *defined terms* which are based off of our agreed upon undefined terms.

The **line segment**, or **segment**, between points A and B on $\overleftrightarrow{AB}$, called $\overline{AB}$, consists of the **endpoints** A and B and all points on $\overleftrightarrow{AB}$ between A and B. Notice that there are no arrows on segments.

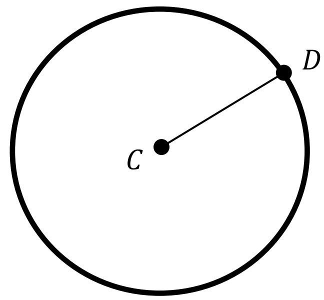


Segment $\overline{AB}$ or $\overline{BA}$.

The **length of a segment $\overline{AB}$** is the distance from A to B. We will deal more with this later. For now, it is denoted simply as the letters AB. Notice there is no extra notation. The length of the segment is different than the segment itself, because the length of a segment is a number.

A **radius** is a segment from the center of a circle to a point on the circle.

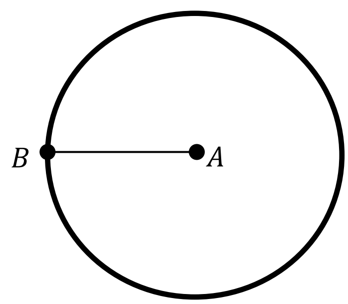
Given a point C in a plane and a number $r > 0$, the **circle** with center C and radius r is the set of all points in the plane that are distance r from point C.



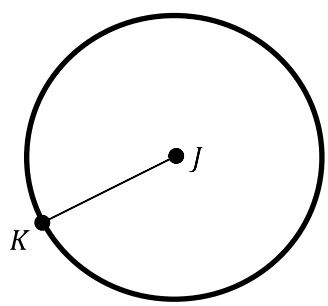
This circle is $\odot C$, with radius CD.

BASIC CONSTRUCTIONS – CIRCLES

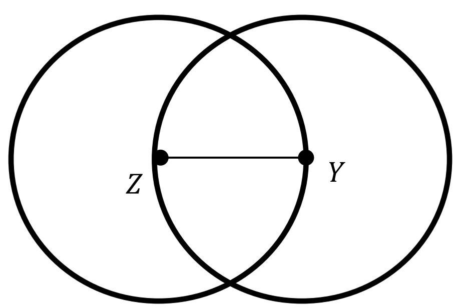
- ⊙A, radius AB



- ⊙J, radius JK



- ⊙Z, with radius ZY and ⊙Y, with radius YZ. Students will be given this challenge problem and notice the difference between the two radii.

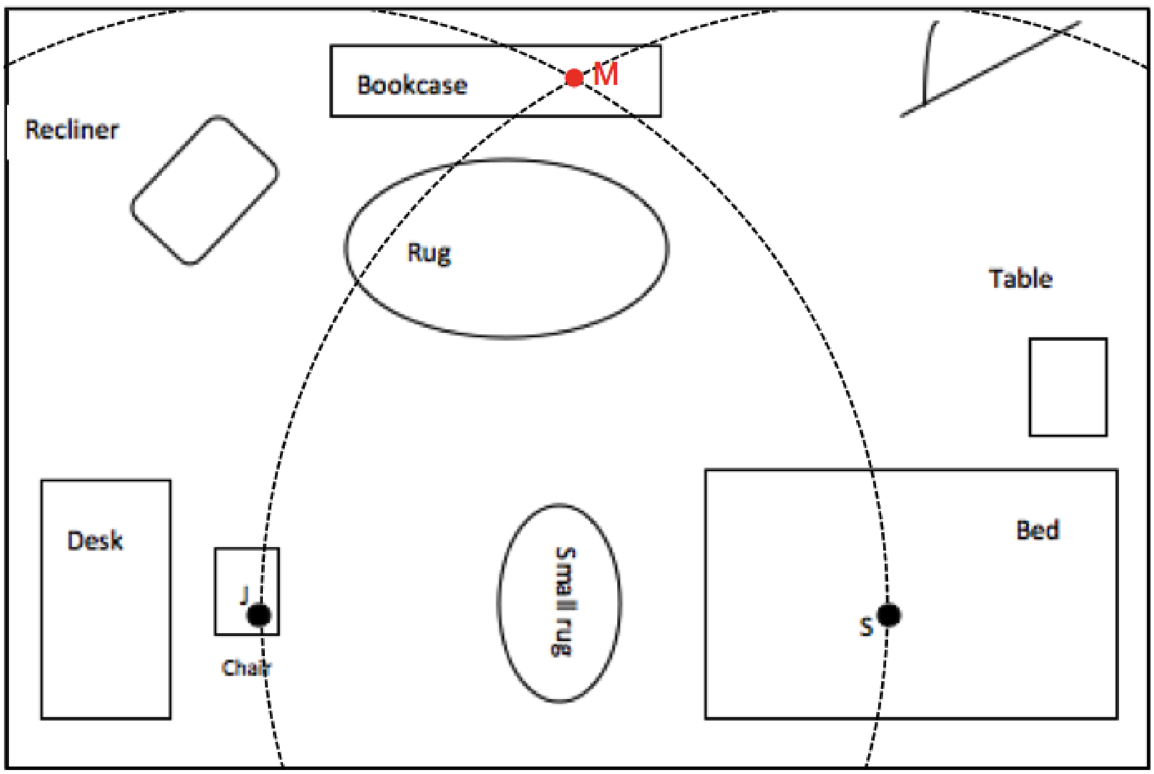


BASIC CONSTRUCTIONS – EQUILATERAL TRIANGLE

Student Activity 1: Sitting Cats

You need a compass and a straightedge.

Margie has three cats. She has heard that cats in a room position themselves at equal distances from one another and wants to test that theory. Margie notices that Simon, her tabby cat, is in the center of her bed (at S), while JoJo, her Siamese, is lying on her desk chair (at J). If the theory is true, where will she find Mack, her calico cat? Use the scale drawing of Margie's room shown below, together with (only) a compass and straightedge. Place an M where Mack will be if the theory is true.



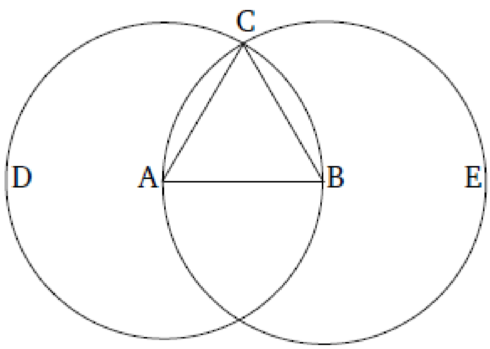
Mathematical Modeling Exercise: Euclid, Proposition 1

Euclid, a very influential mathematician from Greece, shaped much of what we know about Geometry. He looked at the type of problem we just observed—the construction of an equilateral triangle. He did this, with only a compass and straight edge, in 300 BCE. Let's see how Euclid approached this problem. Look at his first proposition, and compare his steps with yours.

Proposition 1

In this margin, compare your steps with Euclid's.

To construct an equilateral triangle on a given finite straight-line.



Let AB be the given finite straight-line.

So it is required to construct an equilateral triangle on the straight-line AB.

Let the circle BCD with center A and radius AB have been drawn [Post. 3], and again let the circle ACE with center B and radius BA have been drawn [Post. 3]. And let the straight-lines CA and CB have been joined from the point C, where the circles cut one another,† to the points A and B (respectively) [Post. 1].

And since the point A is the center of the circle CDB, AC is equal to AB [Def. 1.15]. Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 1.15]. But CA was also shown (to be) equal to AB. Thus, CA and CB are each equal to AB. But things equal to the same thing are also equal to one another [C.N. 1]. Thus, CA is also equal to CB. Thus, the three (straight-lines) CA, AB, and BC are equal to one another.

Thus, the triangle ABC is equilateral, and has been constructed on the given finite straight-line AB. (Which is) the very thing it was required to do.

FINAL VOCABULARY

A **geometric construction** is a set of instructions for drawing points, lines, circles, and figures in the plane.

The two most basic types of instructions are the following:

1) Given any two points A and B, a straightedge can be used to draw the line $\overleftrightarrow{AB}$ or segment $\overline{AB}$.
2) Given any two points C and B, use a compass to draw the circle that has its center at C that passes through B. (Abbreviation: Draw $\odot C$; center C, radius CB.

Constructions also include steps in which the points where lines or circles intersect are selected and labeled.

A **figure** is a set of points in a plane. Usually, the term refers to certain common shapes as triangles, squares, rectangles, etc. However, the definition is broad enough to include any set of points, so a triangle with a line segment sticking out of it is also a figure.

An **equilateral triangle** is a triangle with all sides of equal length.