Number Sets

1. Natural Number – a counting number. Abbreviated as $\mathbb{N}$.
Ex. $\{1, 2, 3, 4, 5, \dots\}$

Natural numbers have an inherent property of either being *prime* or *composite*.

A **prime number** is a number whose *factors* are 1 and itself. A factor is a number when, multiplied to another number, yields the original number.

For instance, 7 is a prime number because it has two factors: 1 and 7. 1 x 7 = 7.

A **composite number** is not a prime number. So, 6 is not a prime number because it has four factors: 1, 2, 3, and 6.

2. Whole Number – the natural numbers, but with 0 in the list. Abbreviated as $\mathbb{W}$.
Ex. {0, 1, 2, 3, 4, 5, …}

3. Integer – positives and negatives. Basically, the whole numbers, but also including their negatives. Abbreviated as $\mathbb{Z}$, because of the German word for numbers, "Zahlen."
Ex. {…, -3, -2, -1, 0, 1, 2, 3, …}

4. Rational number – The formal definition is that it is a number written as a ratio of $\frac{p}{q}$, where p and q are both integers. Abbreviated as $\mathbb{Q}$, for quotient. Basically, it's a number that is a fraction.

Three different ways to see rational numbers:

i) *Fractions*

Ex. $\frac{1}{2}$: 1 and 2 are both integers and they are written as a ratio.

Ex. $-\frac{3}{7}$: –3 and 7 are both integers and they are written as a ratio.

Ex. $\frac{5}{1}$: 5 and 1 are both integers and they are written as a ratio.

ii) *Terminating decimals* (decimals that stop) are also rational numbers. It's possible to rewrite a terminating decimal as a fraction.

Ex. $0.5 = \frac{1}{2}$, and 1 and 2 are both integers written as a ratio.

Ex. -3.81 is a terminating decimal.

Ex. 14.772 is a terminating decimal.

iii) *Repeating decimals* (decimals that go on forever in a pattern) can be written as fractions.

Ex. $0.3333333333 \ldots \frac{1}{3}$

Ex. $0.\overline{1} = \frac{1}{9}$

Ex. $-0.\overline{142857} = \frac{1}{7}$

Note: You do not have to turn every rational number into a fraction. Just knowing these three cases is enough to identify a rational number!

5. Irrational Number – Numbers that can't be written as a fraction. Literally means "not rational."

An irrational number:

1) Cannot be written as a fraction.
2) Cannot be a terminating decimal.
3) Cannot be a repeating decimal.

Ex. $\pi, \sqrt{2}, \sqrt{3}, 2\pi$

6. Real Number – Any number! Abbreviated as $\mathbb{R}$.

Putting it all together:

- A number can be in more than one set. For instance, the number $\frac{5}{12}$ is in $\mathbb{Q}$ and $\mathbb{R}$.

  Ex. 4: $\mathbb{N}, \mathbb{W}, \mathbb{I}, \mathbb{Q}, \mathbb{R}$

  Ex. –13: $\mathbb{I}, \mathbb{Q}, \mathbb{R}$

  Ex. $\sqrt{17}$: Irrational, $\mathbb{R}$

- Operations (addition, subtraction, multiplication, division, exponents) can affect what set a number is in.

Ex. $\frac{2}{5}+\frac{3}{5}$ While both numbers are rational, if you add them together, you get $\frac{5}{5}=1$. Therefore, it is natural, whole, rational, and real.

Ex. $\pi+5$ Because π is an irrational number and 5 is a natural number, adding them together will get you another irrational number.

Guided Practice:

Identify all of the sets the following numbers are in:

1. $-\frac{5}{16}$ $(\mathbb{Q}, \mathbb{R})$
2. 83 $(\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R})$
3. 0 $(\mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R})$
4. $5-\sqrt{45}$ (Irrational, $\mathbb{R}$)
5. $10-5.266$ $(\mathbb{Q}, \mathbb{R})$

There are some numbers that don't belong in any set—if we ever see something like $\sqrt{-4}$, it's not possible to do. So it doesn't belong to a set.

Important: There is a difference between $\sqrt{-4}$ and $-\sqrt{4}$. The first one has no solution; the second one is -2.